

CS 201 Homework 4

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August 27, 2026

Question 1.

Prove that if n is an odd integer, then n^2 is odd.

Write $n = 2k + 1$ for some integer k . Then

$$n^2 = (2k + 1)^2 = 4k^2 + 4k + 1 = 2(2k^2 + 2k) + 1,$$

which is of the form $2m + 1$ with $m = 2k^2 + 2k$ an integer, so n^2 is odd.

Question 2.

Let A and B be sets. Show that $|A \cup B| = |A| + |B| - |A \cap B|$.

Every element of $A \cup B$ is counted once in $|A| + |B|$ unless it lies in both sets, in which case it is counted twice — once for A and once for B . Subtracting $|A \cap B|$ removes exactly that double count, leaving each element of $A \cup B$ counted once.

Recurrence Relation

Solve the recurrence $T(n) = 2T(n/2) + n$ with $T(1) = 1$.

By the Master Theorem with $a = 2$, $b = 2$, $f(n) = n$: since $f(n) = \Theta(n^{\log_b a}) = \Theta(n)$, this is case 2, so $T(n) = \Theta(n \log n)$.

Question 4.

Use the `arabicparts` environment for sub-parts that should include the question number.

4.1) Show $\lceil x \rceil - \lfloor x \rfloor \in \{0, 1\}$ for any real x .

4.2) Give an x for which this is 0, and one for which it is 1.

4.1) If x is an integer, $\lceil x \rceil = \lfloor x \rfloor = x$, so the difference is 0. Otherwise $\lfloor x \rfloor < x < \lceil x \rceil = \lfloor x \rfloor + 1$, so the difference is 1.

4.2) $x = 3$ gives 0; $x = 3.5$ gives 1.

Question 5.

Use the `alphaparts` environment for lettered sub-parts, and prove the following by induction.

(a) State the claim: $\sum_{k=1}^n (2k - 1) = n^2$.

(b) Prove it.

Base case ($n = 1$): $\sum_{k=1}^1 (2k - 1) = 1 = 1^2$.

Inductive step: Assume $\sum_{k=1}^n (2k - 1) = n^2$. Then

$$\sum_{k=1}^{n+1} (2k - 1) = n^2 + (2(n+1) - 1) = n^2 + 2n + 1 = (n+1)^2,$$

completing the induction.

Question 8.

This question is forced to start at number 8 — useful when a handout's numbering does not match the order questions appear in this document.

