

Algorithms: Homework #3

Due on October 2, 2026 at 11:59pm

Professor Ada Lovelace Section B

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Problem 1

Show that $f(n) = 3n^2 + 5n + 2$ is $\mathcal{O}(n^2)$ by giving an explicit positive constant c and threshold n_0 such that $f(n) \leq c \cdot n^2$ for all $n \geq n_0$.

Solution

Take $n \geq 1$, so $n \leq n^2$ and $1 \leq n^2$. Then

$$\begin{aligned} 3n^2 + 5n + 2 &\leq 3n^2 + 5n^2 + 2n^2 \\ &= 10n^2. \end{aligned}$$

So $c = 10$ and $n_0 = 1$ satisfy the definition, and $f(n) \in \mathcal{O}(n^2)$.

Problem 2

Let $\Sigma = \{a, b\}$. Draw a DFA that accepts every string containing the substring **ab**.

Solution

The automaton needs three states: q_0 (no progress toward **ab**), q_1 (just saw an **a**), and q_2 (accepting; **ab** has been seen).

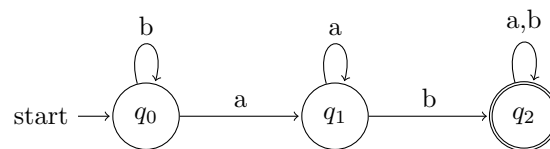


Figure 1: DFA accepting strings containing **ab**

Once the machine reaches q_2 it never leaves, so it accepts exactly the strings containing **ab** as a substring.

Problem 3

Write pseudocode for **Binary-Search**($A, target$) over a sorted array A , returning an index or -1 if $target$ is absent.

Solution

```
1: function BINARY-SEARCH( $A, target$ )
2:    $lo \leftarrow 0, hi \leftarrow \text{length}(A) - 1$ 
3:   while  $lo \leq hi$  do
4:      $mid \leftarrow \lfloor (lo + hi)/2 \rfloor$ 
5:     if  $A[mid] = target$  then
6:       return  $mid$ 
7:     else if  $A[mid] < target$  then
8:        $lo \leftarrow mid + 1$ 
9:     else
10:       $hi \leftarrow mid - 1$ 
11:    end if
12:  end while
13:  return  $-1$ 
14: end function
```

Algorithm 1: Binary search over a sorted array

Problem 4

Prove that $\sum_{k=1}^n k = \frac{n(n+1)}{2}$ by induction on n .

Proof. **Base case** ($n = 1$): the left side is 1, and the right side is $\frac{1 \cdot 2}{2} = 1$, so the claim holds.

Inductive step: assume $\sum_{k=1}^n k = \frac{n(n+1)}{2}$ for some $n \geq 1$. Then

$$\sum_{k=1}^{n+1} k = \sum_{k=1}^n k + (n+1) = \frac{n(n+1)}{2} + (n+1) = \frac{(n+1)(n+2)}{2},$$

which is exactly the claimed formula with n replaced by $n+1$. By induction, the identity holds for every $n \geq 1$. $\square$

Problem 7

Evaluate $\int_0^1 (3x^2 - 2x) \, dx$.

Solution

$$\int_0^1 (3x^2 - 2x) \, dx = [x^3 - x^2]_0^1 = 1 - 1 = 0.$$